On the Ground State of Two Flavor Color Superconductor

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The diquark condensate susceptibility in neutral color superconductor at moderate baryon density is calculated in the frame of two flavor Nambu-Jona-Lasinio model. When color chemical potential is introduced to keep charge neutrality, the diquark condensate susceptibility is negative in the directions without diquark condensate in color space, which may be regarded as a signal of the instability of the conventional ground state with only diquark condensate in the color 3 direction.

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It is generally accepted that, the cold and dense quark matter favors the formation of diquark condensate and is in the superconducting phase[1], which may be realized in compact stars and, in very optimistic cases, even in heavy-ion collisions. To have a stable and macroscopic color superconductor, one should take into account the electric and color charge neutrality condition[2, 3] which leads to a new phase, the gapless color superconductivity[4] or the breached pairing phase [5]. In this new phase, the most probable temperature is finite but not zero[6], and the Meissner screening masses squared can be negative [7]. In two flavor case, the color neutrality can be satisfied by introducing a color chemical potential μ_8 in the four-fermion interaction theory at moderate baryon density[3], or by a dynamic generation of a condensation of gluon field A_0^8 in the frame of perturbative QCD at extremely high baryon density [8] where the back ground gluon field $\langle A_0^8 \rangle$ plays the role of the color chemical potential μ_8 .

In conventional approach to investigating two flavor color superconductivity, one chooses the first two colors participating in the diquark condensate and let the third one be free[9]. When the Lagrangian in the study is color SU(3) symmetric, the ground states with different color breaking directions can be connected by some SU(3) transformations in color space and hence considering only one direction is enough to describe the system. However, when the color-charge neutrality is taken into account, one has to introduce nonzero color chemical potentials at moderate baryon density in the Nambu-Jona-Lasinio(NJL) type model, and the global color SU(3)symmetry of the NJL model is then explicitly broken. In this case the state with only two colors participating in the Cooper pairing is probably not the ground state, and one has to calculate thermodynamical potentials for different states and chooses the one with the minimum thermodynamical potential as the ground state. In this paper, we will calculate the diquark condensate susceptibility in the other two directions without diquark condensate in color space, from which we can judge if the conventional ground state is stable when the color charge neutrality in considered.

We work in the widely used NJL model applied to quarks[10]. The model has been successfully used to study chiral symmetry restoration[10, 11], isospin

symmetry spontaneously broken[12, 13], and color superconductivity[3, 4, 6, 14, 15, 16, 17, 18, 19] at moderate baryon density. In chiral limit, the flavor SU(2) Lagrangian density including quark-quark interaction sector is defined as

$$\mathcal{L} = \bar{\psi} \left(i \gamma^{\mu} \partial_{\mu} + \mu \gamma_{0} \right) \psi + G_{S} \left(\left(\bar{\psi} \psi \right)^{2} + \left(\bar{\psi} i \gamma_{5} \vec{\tau} \psi \right)^{2} \right)$$

$$+ G_{D} \left(\bar{\psi}_{i\alpha}^{c} i \gamma^{5} \epsilon^{ij} \epsilon^{\alpha\beta\gamma} \psi_{j\beta} \right) \left(\bar{\psi}_{i\alpha} i \gamma^{5} \epsilon^{ij} \epsilon^{\alpha\beta\gamma} \psi_{j\beta}^{c} \right) , (1)$$

where G_S and G_D are, respectively, coupling constants in color singlet and anti-triplet channels, $\psi^c = C\bar{\psi}^T$ and $\bar{\psi}^c = \psi^T C$ are charge-conjugate spinors, $C = i\gamma^2\gamma^0$ is the charge conjugation matrix, the quark field $\psi_{i\alpha}$ with flavor index i and color index α is a flavor doublet and color triplet as well as a four-component Dirac spinor, $\tau = (\tau_1, \tau_2, \tau_3)$ are Pauli matrices in flavor space, and ϵ^{ij} and $\epsilon^{\alpha\beta\gamma}$ are, respectively, totally antisymmetric tensors in flavor and color spaces. We focus in the following on the color symmetry breaking phase with nonzero diquark condensates defined as

$$\Delta_{\gamma} = -2G_D \langle \bar{\psi}^c_{i\alpha} i \gamma^5 \epsilon^{ij} \epsilon^{\alpha\beta\gamma} \psi_{j\beta} \rangle, \qquad \gamma = 1, 2, 3 \ . \tag{2}$$

To simplify the calculation, we assume that the chiral symmetry is already restored in this phase. This assumption is confirmed when the coupling constant G_D in the diquark channel is not too large[15]. To ensure color and electric neutralities, one should introduce a set of color chemical potentials $\mu_a(a=1,2,...,8)$ with respect to color charges $Q_1,Q_2,...,Q_8$ and an electric chemical potential μ_e with respect to the electric charge (electrons are included). The ground state of the system is determined by the gap equations which can be obtained by minimizing the thermodynamical potential Ω ,

$$\partial \Omega / \partial \Delta_{\gamma} = 0 , \qquad (3)$$

and the charge neutrality condition,

$$Q_e = -\partial\Omega/\partial\mu_e = 0,$$

$$Q_a = -\partial\Omega/\partial\mu_a = 0, \quad (a = 1, \dots, 8).$$
 (4)

The above twelve coupled equations (3) and (4) determine self-consistently the physical condensates Δ_{γ} and the chemical potentials μ_e and μ_i as functions of temperature T and baryon chemical potential μ_b .

We first consider the conventional ground state with

$$\Delta_1 = \Delta_2 = 0 , \quad \Delta_3 \equiv \Delta \neq 0 . \tag{5}$$

In this case, Q_1, \dots, Q_7 vanishes automatically, and only $Q_8 \neq 0$. Therefore, we can only introduce the color chemical potential μ_8 with respect to the color charge Q_8 to ensure color neutrality, and the quark chemical potential matrix

$$\mu = diag(\mu_{u1}, \mu_{u2}, \mu_{u3}, \mu_{d1}, \mu_{d2}, \mu_{d3}) \tag{6}$$

in color and flavor space can be expressed in terms of the baryon chemical potential μ_b , electrical chemical potential μ_e , and color chemical potentials μ_8 ,

$$\mu_{u1} = \mu_{u2} = \mu_b/3 - 2\mu_e/3 + \mu_8/3 ,$$

$$\mu_{u3} = \mu_b/3 - 2\mu_e/3 - 2\mu_8/3 ,$$

$$\mu_{d1} = \mu_{d2} = \mu_b/3 + \mu_e/3 + \mu_8/3 ,$$

$$\mu_{d3} = \mu_b/3 + \mu_e/3 - 2\mu_8/3 .$$
(7)

Generally, μ_8 is nonzero, and the color symmetry of the NJL Lagrangian is explicitly broken down to $SU(2) \bigotimes U(1)$ with generators T_1, T_2, T_3 and T_8 . In mean field approximation the thermodynamical potential Ω_0 of the system can be evaluated as

$$\Omega_0 = \frac{\Delta^2}{4G_D} - \frac{T}{2} \sum_{r} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} Tr \ln \mathcal{S}_{mf}^{-1} + \frac{\mu_e^4}{12\pi^2} , \quad (8)$$

where the last term is the contribution from the electron gas. In the modified 12-dimensional Nambu-Gorkov space with color and flavor indices, defined by the field vector

$$\bar{\Psi} = (\bar{\psi}_{u1}, \ \bar{\psi}_{d2}^c, \ \bar{\psi}_{d2}, \ \bar{\psi}_{u1}^c, \ \bar{\psi}_{d1}, \ \bar{\psi}_{u2}^c,
\bar{\psi}_{u2}, \ \bar{\psi}_{d1}^c, \ \bar{\psi}_{u3}, \ \bar{\psi}_{u3}^c, \ \bar{\psi}_{d3}, \ \bar{\psi}_{d3}^c) ,$$
(9)

the mean field quark propagator \mathcal{S}_{mf}^{-1} is diagonal and can be expressed as

$$S_{mf}^{-1} = diag \left(S_{u1,d2}^{-\Delta} \ S_{d2,u1}^{-\Delta} \ S_{d1,u2}^{\Delta} \ S_{u2,d1}^{\Delta} \ S_{u3,u3}^{0} \ S_{d3,d3}^{0} \right)$$
(10)

where the diagonal blocks are defined as

$$S_{i\alpha,j\beta}^{\chi} = \begin{pmatrix} [\mathcal{G}_0^+]_{i\alpha}^{-1} & i\gamma_5\chi\\ i\gamma_5\chi & [\mathcal{G}_0^-]_{j\beta}^{-1} \end{pmatrix}$$
(11)

with the free quark propagators

$$\left[\mathcal{G}_0^{\pm}\right]_{i\alpha}^{-1} = \left(i\omega_n\gamma_0 - \mathbf{p}\cdot\gamma \pm \mu_{i\alpha}\gamma_0\right) . \tag{12}$$

Since the quark propagator S_{mf}^{-1} is diagonal, we can analytically take its trace in the color, flavor and Dirac spaces and make the Matsubara frequency summation, the thermodynamic potential can be expressed as an explicit function of $\Delta, \mu_e, \mu_8, \mu_b$ and T. Minimizing the known Ω_0 , we then obtain[3] in the color symmetry spontaneously breaking phase the gap equation

$$1 - 8G_D \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \sum_{\epsilon = +} \frac{1 - f(E_+^{\epsilon}) - f(E_-^{\epsilon})}{E_{\Delta}^{\epsilon}} = 0 \quad (13)$$

and the charge neutrality condition

$$Q_{8} = \int \frac{d^{3}\mathbf{p}}{(2\pi)^{3}} \sum_{\epsilon=\pm} \epsilon \left[\frac{E_{0}^{\epsilon}}{E_{\Delta}^{\epsilon}} \left(1 - f(E_{+}^{\epsilon}) - f(E_{-}^{\epsilon}) \right) + (f(E_{u3}^{\epsilon}) + f(E_{d3}^{\epsilon})) \right] = 0 ,$$

$$Q_{e} = \int \frac{d^{3}\mathbf{p}}{(2\pi)^{3}} \sum_{\epsilon=\pm} \left[\epsilon \frac{E_{0}^{\epsilon}}{E_{\Delta}^{\epsilon}} \left(1 - f(E_{+}^{\epsilon}) - f(E_{-}^{\epsilon}) \right) + 3 \left(f(E_{+}^{\epsilon}) - f(E_{-}^{\epsilon}) \right) - \epsilon \left(2f(E_{u3}^{\epsilon}) - f(E_{d3}^{\epsilon}) \right) \right] - \frac{\mu_{e}^{3}}{3\pi^{2}} = 0 ,$$

$$(14)$$

where the quasi-particle energies are defined as $E_{\mp}^{\pm}=E_{\Delta}^{\pm}\mp\delta\mu, E_{\Delta}^{\pm}=\sqrt{(|\mathbf{p}|\pm\bar{\mu})^2+\Delta^2}, E_0^{\pm}=|\mathbf{p}|\pm\bar{\mu}, E_{u3}^{\pm}=|\mathbf{p}|\pm\mu_{u3}$ and $E_{d3}^{\pm}=|\mathbf{p}|\pm\mu_{d3}$ with the two effective chemical potentials $\bar{\mu}$ and $\delta\mu$ given by $\bar{\mu}=\mu_b/3-\mu_e/6+\mu_8/3$ and $\delta\mu=\mu_e/2$, and $f(x)=1/\left(e^{x/T}+1\right)$ is the Fermi-Dirac distribution function. The equations (13) and (14) determine simultaneously the order parameter Δ and chemical potentials μ_e and μ_8 in the conventional ground state of neutral color superconductor.

Now we come to the question whether the conventional ground state defined through (5) is stable. To answer this question we calculate the second order derivation of the thermodynamical potential Ω with respect to the diquark condensates Δ_1 and Δ_2 ,

$$\kappa = \frac{\partial^2 \Omega}{\partial \Delta_1^2} \Big|_{\Delta_1 = \Delta_2 = 0} = \frac{\partial^2 \Omega}{\partial \Delta_2^2} \Big|_{\Delta_1 = \Delta_2 = 0}.$$
 (15)

We call the quantity κ diquark condensate susceptibility.

Since the explicit form of Ω with finite condensates Δ_1 and Δ_2 is not easy to obtained, we shall use the method of perturbation. We take

$$\Delta_1 = \delta_1 \ll \Delta ,
\Delta_2 = \delta_2 \ll \Delta ,$$
(16)

and apply the Taylor expansion

$$\Omega_{\delta} - \Omega_0 = \frac{1}{2} \kappa \left(\delta_1^2 + \delta_2^2 \right) + \dots \,. \tag{17}$$

Here the perturbed thermodynamical potential Ω_{δ} can be written as

$$\Omega_{\delta} = \frac{\Delta^2 + \delta_1^2 + \delta_2^2}{4G_D} + \frac{\mu_e^4}{12\pi^2} - \frac{T}{2} \sum_{n} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} Tr \ln \left[\mathcal{S}_{mf}^{-1} + \Gamma \left(\delta_1, \delta_2 \right) \right] (18)$$

with the matrix $\Gamma(\delta_1, \delta_2)$ defined as

Taking into account the expansion

$$Tr\ln(A^{-1}+B) = Tr\ln A^{-1} - \sum_{n=1}^{\infty} \frac{(-1)^n}{n} Tr(AB)^n (20)$$

for general matrices A and B and the relation

$$Tr\left[\mathcal{S}_{mf}\Gamma\left(\delta_{1},\delta_{2}\right)\right]=0, \qquad (21)$$

which ensures the linear term to be zero, we can expand Ω_{δ} to the quadratic terms in δ_1 and δ_2 , and obtain the diquark condensate susceptibility

$$\kappa = \frac{1}{2G_D} + \frac{T}{4} \sum_{n} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} Tr\left(\mathcal{S}_{mf} \Gamma(1,0)\right)^2 . \quad (22)$$

The next task is to calculate κ based on the known Δ, μ_e and μ_8 in the conventional ground state. After a somewhat complicated but straightforward algebra calculation and using the gap equation (13), κ can be simplified as

$$\kappa = -2\mu_8 K(0) \tag{23}$$

with the function K(x) given by

$$K(x) = \int \frac{d^{3}\mathbf{p}}{(2\pi)^{3}} \sum_{\epsilon = \pm} \left[\left(\frac{1}{F_{1}^{\epsilon}} - \frac{1}{F_{2}^{\epsilon}} \right) \frac{f(E_{u3}^{\epsilon}) + f(E_{d3}^{\epsilon}) - 1}{E_{\Delta}^{\epsilon}} + \left(\frac{1}{F_{1}^{\epsilon}} + \frac{1}{F_{2}^{\epsilon}} \right) \frac{f(E_{+}^{\epsilon}) + f(E_{-}^{\epsilon}) - 1}{E_{\Delta}^{\epsilon}} \right],$$
(24)

where the x-dependence is hidden in $F_{1,2}^{\pm}$ defined as

$$F_1^{\pm}(x, |\mathbf{p}|) = x + \mu_8 \mp E_0^{\pm} \mp E_{\Delta}^{\pm} ,$$

$$F_2^{\pm}(x, |\mathbf{p}|) = x + \mu_8 \mp E_0^{\pm} \pm E_{\Delta}^{\pm} .$$
 (25)

It is clear that, in the case without considering the color chemical potential, namely $\mu_8=0$, we have $\kappa=0$, the conventional ground state is stable under the perturbation. In fact, when $\mu_8=0$ the thermodynamic potential depends only on the quantity $\sqrt{\Delta_1^2+\Delta_2^2+\Delta_3^2}$,

thus one can choose $\Delta_1 = \Delta_2 = 0, \Delta_3 \neq 0$ without losing generality. When explicitly considering $\mu_8 \neq 0$, the function K(x) is related to the color charge density Q_8 through [18]

$$K(-\mu_8) = 2\frac{Q_8}{\Delta^2} = 0 ,$$
 (26)

we can then expand K(x) around $x = -\mu_8$. Since the magnitude of μ_8 is only a few MeV in the whole color breaking phase[3, 18], which is much less than the order parameter Δ , the baryon chemical potential μ_b , and the momentum cutoff Λ , we can keep only the linear term in the expansion,

$$\kappa = -2\mu_{8}^{2}K'(-\mu_{8})$$

$$= -\frac{4\mu_{8}^{2}}{\Delta^{4}} \int \frac{d^{3}\mathbf{p}}{(2\pi)^{3}} \sum_{\epsilon=\pm} \left[\frac{(E_{0}^{\epsilon})^{2} + (E_{\Delta}^{\epsilon})^{2}}{E_{\Delta}^{\epsilon}} \right]$$

$$\left(1 - f(E_{+}^{\epsilon}) - f(E_{-}^{\epsilon})\right) - 2E_{0}^{\epsilon} \left(1 - f(E_{u3}^{\epsilon}) - f(E_{d3}^{\epsilon})\right) \right]$$

$$< -\frac{4\mu_{8}^{2}}{\Delta^{4}} \int \frac{d^{3}\mathbf{p}}{(2\pi)^{3}} \sum_{\epsilon=\pm} \left[\frac{(E_{\Delta}^{\epsilon} - (E_{3u}^{\epsilon} + E_{3d}^{\epsilon})/2)^{2}}{E_{\Delta}^{\epsilon}} \right]$$

$$\left(1 - f(E_{+}^{\epsilon}) - f(E_{-}^{\epsilon})\right) \right]$$

$$< 0. \tag{27}$$

Therefore, we have proven analytically that, under the condition $|\mu_8| \ll \Delta, \mu_b, \Lambda, \kappa$ is negative in the conventional ground state when the color charge neutrality is considered. This conclusion can also be proven numerically. To do numerical calculation, we choose the parameters $G_S = 5.01 GeV^{-2}, \Lambda = 0.653 GeV$ and $G_D = 3G_S/4$ as in Ref.[3]. With these parameters the minimum baryon chemical potential where the color superconductivity phase starts is $\mu_b/3 = 330$ MeV. The baryon chemical potential dependence of μ_8 and κ , calculated through (13), (14) and (23), is shown in Fig.1 in the case with both color and electrical chemical potentials and in Fig.2 with only color chemical potential (namely taking $\mu_e = 0$). We see that, κ is negative in the whole color breaking phase, independent of whether

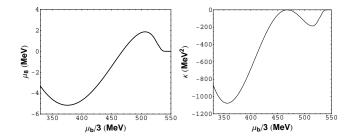


FIG. 1: The color chemical potential μ_8 and κ defined in (23) as functions of baryon chemical potential μ_b at temperature T=0 for color superconductor with both constraints $Q_8=0$ and $Q_e=0$.

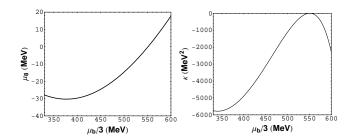


FIG. 2: The color chemical potential μ_8 and κ defined in (23) as functions of baryon chemical potential μ_b at temperature T=0 for color superconductor with only the constraint $Q_8=0$.

the electrical charge neutrality is taken into account or not. In fact, in both cases with and without electrical charge neutrality, μ_8 is really very small, compared with Δ, μ_b, Λ which are all the order of hundreds MeV, the assumption used in the analytic calculation is safe.

In summary, we have investigated the relation between the explicit color symmetry breaking and the ground state which reflects the spontaneous color symmetry breaking. For color superconductor without color neutrality, the Lagrangian of the system is color SU(3) symmetrical, and the spontaneous breaking can happen in only one of the three directions in color space, namely from SU(3) to SU(2). However, when the color chemical potential is taken into account, the Lagrangian of NJL model loses its color SU(3) symmetry explicitly, and therefore the spontaneous color breaking in only one direction (Δ_3) may be impossible. Through calculating the diquark condensate susceptibility κ in the other two directions, we have proven analytically and numerically that κ is always negative in the whole color breaking phase in the frame of flavor SU(2) NJL model. This may be a signal that the conventional ground state in unstable. More detail investigations in this aspect will be done in our future works.

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